Solar-concentration optics

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Contents

This document contains some of the information I needed for our projects on solar concentration. It is by no means complete, and may well contain errors (even though the implementations seem in order).

1 Transmission and reflection

This section follows Chapter 4 from Hecht (1998).

1.1 Definitions

The radiant flux or *irradiance* ($\mathrm{W/m^2}$) is defined as

$$I = \varepsilon v \langle E^2 \rangle, \tag{1}$$

where E^2 is averaged over time, while

$$n_\mathrm{t} \equiv \frac{c}{v}, \quad v^2 = \frac{1}{\varepsilon \mu_0} \quad \text{and hence} \quad \varepsilon v = \frac{n_\mathrm{t}}{c\mu_0}. \tag{2}$$

The *reflectance* is then

$$R \equiv \frac{I_\mathrm{r} A \cos\theta_\mathrm{r}}{I_\mathrm{i} A \cos\theta_\mathrm{i}} = \frac{I_\mathrm{r}}{I_\mathrm{i}} = \frac{n_\mathrm{r}/c\mu_0\, E_{0,r}^2}{n_\mathrm{i}/c\mu_0\, E_{0,i}^2} = \left(\frac{E_{0,r}}{E_{0,i}}\right)^2 \equiv r^2, \tag{3}$$

while the *transmittance* is similarly defined as

$$T \equiv \frac{I_\mathrm{t} A \cos\theta_\mathrm{t}}{I_\mathrm{i} A \cos\theta_\mathrm{i}} = \frac{n_\mathrm{r}/c\mu_0\, E_{0,r}^2 \cos\theta_\mathrm{r}}{n_\mathrm{i}/c\mu_0\, E_{0,i}^2 \cos\theta_\mathrm{i}} = \frac{n_\mathrm{t}\cos\theta_\mathrm{t}}{n_\mathrm{i}\cos\theta_\mathrm{i}} \left(\frac{E_{0,t}}{E_{0,i}}\right)^2 \equiv \frac{n_\mathrm{t}\cos\theta_\mathrm{t}}{n_\mathrm{i}\cos\theta_\mathrm{i}}\, t^2. \tag{4}$$

1.2 Reflectance and transmittance at surface boundaries

For a light ray incident on a surface boundary between media with refractive indices n_i (incident) and n_t (transmitted) and incident and transmitted angles θ_i and θ_t, we first compute the argument for Snell's law:

$$s = \frac{n_\mathrm{i}}{n_\mathrm{t}} \sin\theta_\mathrm{i} \tag{5}$$

If $s > 1$, total internal reflection takes place, so that $R = 1$ and $T = 0$. If not, then the angle of transmission is given by Snell's law:

$$\theta_\mathrm{t} = \arcsin s. \tag{6}$$

The Fresnel reflectance for the perpendicular and parallel polarisations is then given by:

$$\begin{aligned} R_\perp &= \left(\frac{n_\mathrm{i}\,\cos\theta_\mathrm{i} - n_\mathrm{t}\,\cos\theta_\mathrm{t}}{n_\mathrm{i}\,\cos\theta_\mathrm{i} + n_\mathrm{t}\,\cos\theta_\mathrm{t}}\right)^2 &= \frac{\sin^2(\theta_\mathrm{t} - \theta_\mathrm{i})}{\sin^2(\theta_\mathrm{t} + \theta_\mathrm{i})} \\ R_\| &= \left(\frac{n_\mathrm{i}\,\cos\theta_\mathrm{t} - n_\mathrm{t}\,\cos\theta_\mathrm{i}}{n_\mathrm{i}\,\cos\theta_\mathrm{t} + n_\mathrm{t}\,\cos\theta_\mathrm{i}}\right)^2 &= \frac{\tan^2(\theta_\mathrm{t} - \theta_\mathrm{i})}{\tan^2(\theta_\mathrm{t} + \theta_\mathrm{i})} \end{aligned} \tag{7}$$

and the unpolarised reflectance can be computed from

$$R = \frac{R_\perp + R_\|}{2}. \tag{8}$$

The transmittance is simply the radiation that is *not* reflected:

$$T = 1 - R. \tag{9}$$

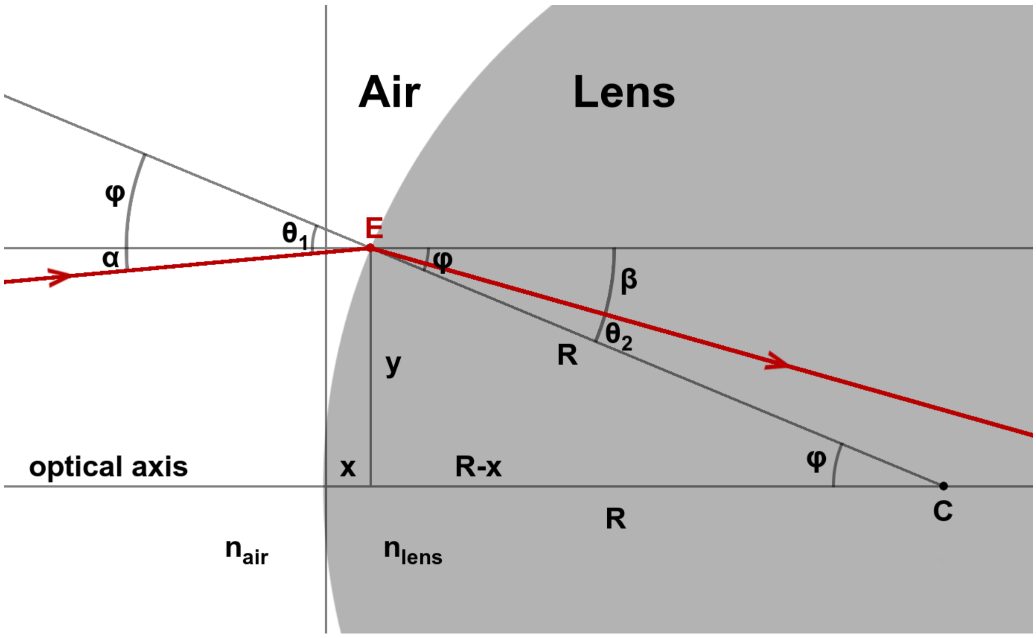

Figure 1: Sketch of a light ray (thick red line with arrows) travelling from left to right and entering a spherical lens (grey background) from air (white background) at entry point E. C denotes the centre of the lens, R the radius of the lens, as well as the normal vector to its surface (since the lens is spherical), with direction φ w.r.t. the optical axis. The angle α is the incoming angle and β the outgoing angle (inside the lens) w.r.t. the *optical axis*, while θ_1 and θ_2 are the incoming and outgoing angles w.r.t. the *normal*. The (vertical) distance of E from the optical axis is indicated with y, while the horizontal distance between the front of the lens (on the optical axis) and E is x. Note that the horizontal line through E has the same orientation as the optical axis.

2 Lens optics

2.1 Spherical lenses

Figure 1 shows a light ray entering a spherical lens from the surrounding medium (air). The refraction upon incidence can be computed as follows, where we will use the subscript label "in" for entry into the lens.

The incoming light ray is specified in two dimensions by the angle α_{in} or $\theta_{\text{in},1}$ and the vertical distance of the entry point y. The lens is completely defined by its (curvature) radius R_{in}. First, we note that E and hence φ_{in}, which is the direction of the normal vector at E, are defined by y_{in}:

$$\sin\varphi_{\text{in}} = \frac{y_{\text{in}}}{R_{\text{in}}}; \quad \tan\varphi_{\text{in}} = \frac{y_{\text{in}}}{R_{\text{in}} - x_{\text{in}}}. \tag{10}$$

Then, Snell's law tells us that

$$\frac{\sin\theta_{\text{in},1}}{\sin\theta_{\text{in},2}} = \frac{\sin(\varphi_{\text{in}} + \alpha_{\text{in}})}{\sin(\varphi_{\text{in}} - \beta_{\text{lens}})} = \frac{n_{\text{lens}}}{n_{\text{air}}}, \tag{11}$$

where n_{med} and n_{lens} are the refraction indices of the surrounding medium and the lens, respectively and the θ's can be computed to determine the reflection upon entrance. From this we can compute β_{lens}, the direction of the light ray inside the lens w.r.t. the optical axis. If the lens is not an actual sphere, we can still use the *curvature radius* R_{in} for the point of entry to compute β_{lens}.

If the lens is a slab of material with (horizontal) thickness d_{lens}, we can compute the vertical position of exit using

$$y_{\text{out}} = y_{\text{in}} + d_{\text{lens}}\ \tan\beta_{\text{lens}} = y_{\text{in}} + \ell_{\text{opt}}\ \sin\beta_{\text{lens}}, \tag{12}$$

with $\ell_{\text{opt}} = \frac{d_{\text{lens}}}{\cos\beta_{\text{lens}}}$ the distance that the light ray travelled inside the lens, which is useful for absorption calculations.

When we mirror Figure 1 in our minds and replace the subscripts “in” with “out”, the initial direction of the light ray is β_{lens}, and the direction of the outgoing ray α_{out} can be computed using the auxiliary angle

$$\sin\varphi_{\text{out}} = \frac{y_{\text{out}}}{R_{\text{out}}}; \quad \tan\varphi_{\text{out}} = \frac{y_{\text{out}}}{R_{\text{out}} - x_{\text{out}}} \tag{13}$$

and

$$\frac{\sin\theta_{\text{in},1}}{\sin\theta_{\text{in},2}} = \frac{\sin(\varphi_{\text{out}} + \alpha_{\text{out}})}{\sin(\varphi_{\text{out}} - \beta_{\text{lens}})} = \frac{n_{\text{lens}}}{n_{\text{air}}}, \tag{14}$$

where the θ's can be used to compute internal reflection.

The total angle of refraction is then given by

$$\Delta\alpha = \alpha_{\text{out}} - \alpha_{\text{in}}, \tag{15}$$

while the vertical ‘jump’ of the ray equals

$$\Delta y = y_{\text{out}} - y_{\text{in}}. \tag{16}$$

2.1.1 Fresnel lenses

For an infinitely thin Fresnel lens, and the light ray entering on the flat side, $d_{\text{lens}} = \ell_{\text{lens}} = 0$, the xs reduce to nought, while

$$R_{\text{in}} = \infty, \tag{17}$$

$$R_{\text{out}} = f_{\text{lens}}\,(n_{\text{lens}} - 1) \tag{18}$$

and

$$y_{\text{out}} = y_{\text{in}}, \quad \Delta y = 0, \tag{19}$$

with f_{lens} the focal length of the lens. Hence, equations 10, 11 and 13 reduce to

$$\varphi_{\text{in}} = 0, \tag{20}$$

$$\sin\beta_{\text{lens}} = -\frac{n_{\text{air}}}{n_{\text{lens}}}\,\sin\alpha_{\text{in}}, \tag{21}$$

and

$$\tan\varphi_{\text{out}} = \frac{y_{\text{in}}}{R_{\text{out}}}. \tag{22}$$

2.2 Designing Fresnel lenses

In order to design a perfect Fresnel lens, one would like to establish R_{out} (or the direction of the local normal vector w.r.t. the optical axis, φ_{out}) as a function of y. When we consider a perpendicularly incoming light ray ($\alpha_{\text{in}} = 0$) at position y_{in}, it should be refracted upon emergence such that it passes through the focal point, so that $\tan\alpha_{\text{out}} = \frac{y_{\text{in}}}{f_{\text{lens}}}$. Snell's law then dictates that, upon emergence,

$$\frac{\sin\theta_{\text{in}}}{\sin\theta_{\text{out}}} = \frac{n_{\text{med}}}{n_{\text{lens}}}. \tag{23}$$

From geometry, we also find that

$$\theta_{\text{out}} = \theta_{\text{in}} + \alpha_{\text{out}} = \varphi_{\text{out}} + \alpha_{\text{out}}. \tag{24}$$

Combining these two equations results in the orientation of the normal vector

$$\tan\varphi_{\text{out}} = \frac{y_{\text{in}}}{\frac{n_{\text{lens}}}{n_{\text{med}}}\sqrt{f^2 + y_{\text{in}}^2} - f}, \tag{25}$$

so that the curvature radius is given by

$$R_{\text{out}} = \frac{n_{\text{lens}}}{n_{\text{med}}}\sqrt{f^2 + y_{\text{in}}^2} - f. \tag{26}$$

2.3 Total internal reflection

The critical (minimum) angle for total internal reflection for an outgoing light ray is given by

$$\sin\theta_\mathrm{c} = \frac{n_\mathrm{med}}{n_\mathrm{lens}}. \tag{27}$$

For a lens made of PMMA, with $n_\mathrm{lens} = 1.4905$ at $\lambda = 589.3\,\mathrm{nm}$, and air ($n_\mathrm{air} = 1.000293$), this angle is

$$\theta_\mathrm{c} = \frac{n_\mathrm{air}}{n_\mathrm{lens}} \approx 42.153°. \tag{28}$$

Note that total internal reflection cannot occur in a plane-parallel sheet of material, since θ_in should exceed 90° in order to obtain $\theta > \theta_\mathrm{c}$ inside the material.

3 Image size and intensity

3.1 Image size for large object distance

For an object at a large distance, *e.g.* the Sun, the image distance equals the focal length. Hence, the two extreme rays from the object through the centre of the lens form an X consisting of straight lines that in turn form two triangles with the same top angle α, the apparent diameter of the object (see Figure 2). Hence, using the apparent *radius* $\alpha/2$,

$$\tan\frac{\alpha}{2} = \frac{d_\mathrm{img}/2}{f_\mathrm{lens}} = \frac{d_\mathrm{img}}{2f_\mathrm{lens}}, \tag{29}$$

so that the image size d_img is given by

$$d_\mathrm{img} = 2f_\mathrm{lens} \cdot \tan\frac{\alpha}{2}. \tag{30}$$

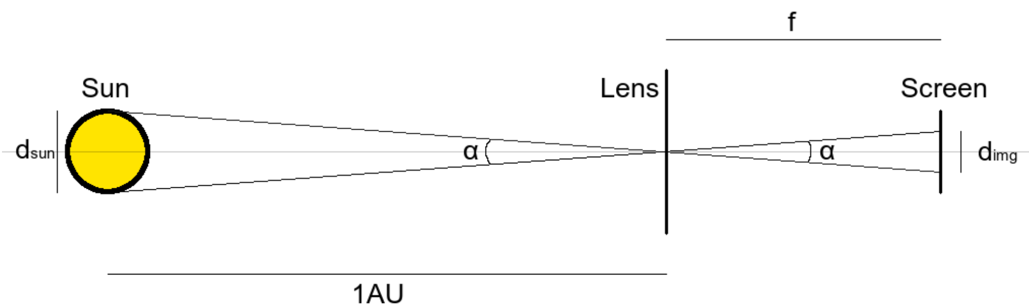

Figure 2: Projection of a Sun image on a screen, by a lens with focal length f.

In the actual case of the Sun, its distance is $\approx 107.5\times$ its diameter,[1] so that the mean apparent diameter of the Sun in the sky is about 0.533°, $\tan\frac{\alpha}{2} \approx 0.00465$ and the average diameter of the solar image is given by

$$d_\mathrm{img} \approx 0.00930\,f_\mathrm{lens}. \tag{31}$$

For a lens with $f_\mathrm{lens} = 30\,\mathrm{cm}$, this amounts to $d_\mathrm{img} \approx 2.79\,\mathrm{mm}$.

3.2 Loss of insolation due to tracking errors

When the tracking of a Solar-concentration module is not perfect, part of the solar image may fall next to the receiver (*e.g.* a solar cell). Assuming that the image runs off a rectangular receiver in only one direction (see Fig. 3a), the amount of sunlight lost can be quantified as follows (van der Sluys et al., 2015).

[1]Hence, as a rule of thumb, the diameter of a solar image is $\sim 1/100$-th of the focal distance of the lens.

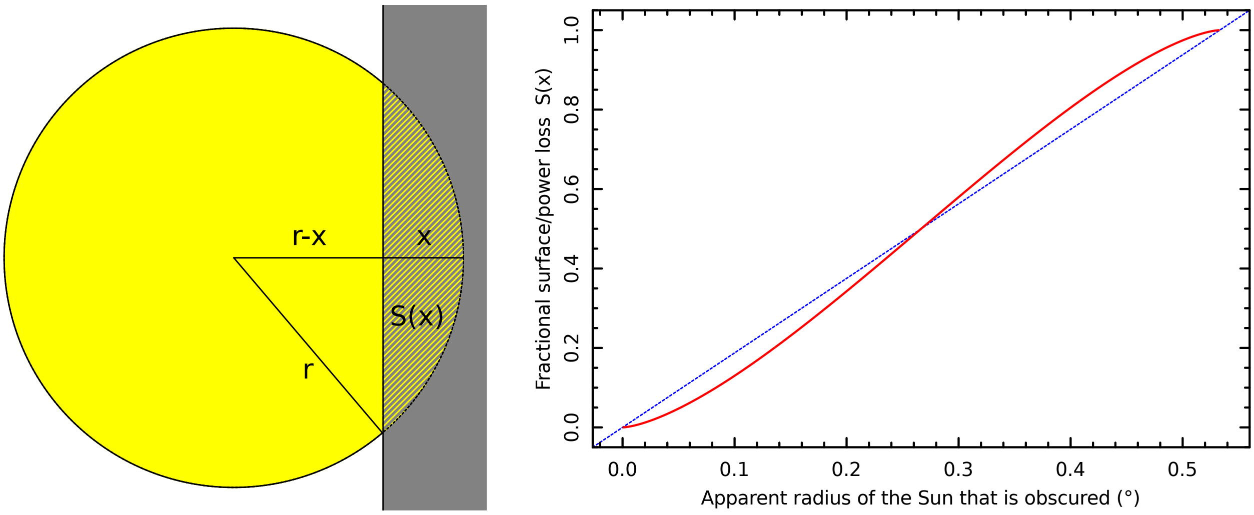

Figure 3: Left panel (a): a sketch of a circular solar image with part of the disc falling outside the receiver (the dark area), with the relevant parameters for Eq. 32; x is the tracking misalignment. Right panel (b): loss of solar power as a function of the tracking error (upper/red solid line), compared to a linear relation (dashes). Initially, the power loss increases slowly with the tracking error, even though an error of 0.07° leads to 8% loss.

The Sun's radius is given by r, and the tracking error x, in the same units (*e.g.* degrees). When we ignore limb darkening on the solar image, the fractional power loss is identical to the fractional surface $S(x)$ of the Sun's disc that falls outside the solar cell:

$$\begin{aligned} S(x) &= \frac{1}{\pi r^2} \int_0^x 2 \cdot \sqrt{r^2 - (r - x')^2} dx' \\ &= \frac{1}{2} + \frac{1}{\pi} \left[\sin^{-1}(f-1) + (f-1)\sqrt{f(2-f)} \right], \end{aligned} \tag{32}$$

where $f \equiv \frac{x}{r}$. The power loss as a function of tracking accuracy is shown in Fig. 3b.

3.3 Maximum concentration factor

Text books sometimes quote a theoretical maximum concentration factor,[2] often without explanation:

$$C \lesssim \frac{1}{\sin^2 \Phi}; \qquad C \lesssim \frac{1}{\sin \Phi}, \tag{33}$$

where Φ is the Sun's apparent radius in the sky (our $\alpha/2$ from Eq. 29), the first equation for spherical lenses or parabolic dishes (2D), and the second the 1D version for linear lenses or parabolic troughs.

The assumption for this is presumably that an optical $f/$-ratio smaller than $\sim$ unity does not often occur. We could state that the radius of a (circular) lens $r_{\rm lens}$ should not exceed its focal length $f_{\rm lens}$:

$$r_{\rm lens} \lesssim f_{\rm lens}. \tag{34}$$

The concentration factor C is defined as:

$$C \equiv \frac{A_{\rm lens}}{A_{\rm img}} = \frac{\pi r_{\rm lens}^2}{\pi (d_{\rm img}/2)^2} = \frac{\pi r_{\rm lens}^2}{\pi \left(f_{\rm lens} \cdot \tan\frac{\alpha}{2}\right)^2} = \frac{r_{\rm lens}^2}{f_{\rm lens}^2 \cdot \tan^2 \Phi} \lesssim \frac{1}{\tan^2 \Phi}, \tag{35}$$

where we used Eqs. 30 and 34 and note that for small angles $\sin\Phi \sim \tan\Phi$ $(\sim \Phi)$. Similar reasoning can lead to the 1D version. Since the Sun's apparent radius is nearly constant, we find that $C \lesssim 46,200$ for the 2D case and $C \lesssim 215$ for the linear case.[3]

[2] More often than one would expect, given the relatively high uselessness of this variable...
[3] Note that $\frac{1\,\mathrm{AU}}{1\,R_\odot} \approx 215$.

3.4 Image intensity and radiation temperature

The flux of light arriving at an image in the focus depends on the flux arriving at the lens and the surface areas of the lens and the image:

$$F_{\rm img} = F_{\rm lens} \frac{A_{\rm lens}}{A_{\rm img}} \tag{36}$$

For the case of the Sun, we find:

$$F_{\rm img} \approx F_{\rm lens} \frac{A_{\rm lens}}{\pi \left(\frac{0.0093 f_{\rm lens}}{2} \right)^2} \approx 1.47 \times 10^4 \; F_{\rm lens} \frac{A_{\rm lens}}{f_{\rm lens}^2}. \tag{37}$$

For $F_{\rm lens} = 1000\,{\rm W/m^2}$, $A_{\rm lens} = 30 \times 30\,{\rm cm^2}$ and $f_{\rm lens} = 30\,{\rm cm}$, the fraction equals unity and the flux at the focus approximates $1.47 \times 10^7\,{\rm W/m^2}$, a concentration factor of 1.47×10^4.

The black-body temperature of such a radiation field, assuming it is in thermal equilibrium by radiation alone, is given by the Stefan-Boltzmann law:

$$F_{\rm img} = \sigma T_{\rm bb}^4, \tag{38}$$

so that

$$T_{\rm bb} = \left(\frac{F_{\rm img}}{\sigma} \right)^{1/4}. \tag{39}$$

For $F_{\rm img} \approx 1.47 \times 10^7$, this yields

$$T_{\rm bb} \approx 4013\,{\rm K}. \tag{40}$$

Hence, adequate cooling is required to limit the temperature.

References

Hecht, E. 1998, Optics, 3rd Ed. (Addison Wesley Longman)

van der Sluys, M., van Kan, P., & Sonneveld, P. 2015, in American Institute of Physics Conference Series, Vol. 1679, American Institute of Physics Conference Series, 080003