Modelling the COP of heat pumps as a function of temperature

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1 A simple functional description of a heat pump

By the first law of thermodynamics, the change in internal energy U of a closed system equals the sum of the heat Q supplied to the system minus the work W done by the system:

$$\Delta U = Q - W. \tag{1}$$

For a heat pump this can be reformulated as:

$$Q_{\rm h} = Q_{\rm c} + W; \qquad W = Q_{\rm h} - Q_{\rm c}. \tag{2}$$

The heat extracted from the cold side Q_C plus the mechanical work W done on the refrigerant is equal to the heat produced on the hot side Q_H. The Coefficient of Performance (COP) is defined as the ratio of heat produced and the work done:

$$\mathrm{COP}_{\rm heating} = \frac{|Q_{\rm h}|}{W} = \frac{Q_{\rm h}}{Q_{\rm h} - Q_{\rm c}}. \tag{3}$$

Under ideal conditions, the entropy S, defined as

$$S = \frac{Q}{T}, \tag{4}$$

is conserved during a heat-pump cycle. Hence the entropy on the hot side $S_{\rm h}$ must be equal to that on the cold side $S_{\rm c}$, so that:

$$\frac{Q_{\rm h}}{T_{\rm h}} = \frac{Q_{\rm c}}{T_{\rm c}}; \qquad Q_{\rm c} = \frac{Q_{\rm h} T_{\rm c}}{T_{\rm h}}. \tag{5}$$

The expression for the (maximal) COP under *isentropic* conditions thus becomes:

$$\mathrm{COP}_{\rm heating} = \frac{Q_{\rm h}}{Q_{\rm h} - \frac{Q_{\rm h} T_{\rm c}}{T_{\rm h}}} = \frac{1}{1 - \frac{T_{\rm c}}{T_{\rm h}}} = \frac{T_{\rm h}}{T_{\rm h} - T_{\rm c}}. \tag{6}$$

Another definition of the COP is the ratio of the heat provided (during an interval) and the (typically electrical) work done, which is equal to the ratio of the heating and electrical powers:

$$\mathrm{COP}_{\mathrm{heating}} \equiv \frac{Q_{\mathrm{h}}}{W} = \frac{\dot{Q}_{\mathrm{H}}}{\dot{W}} \equiv \frac{P_{\mathrm{heat}}}{P_{\mathrm{el}}}. \tag{7}$$

In modelling the functionality of a heat pump, it is recognised that heating power P_{heat}, consumed electrical power P_{el} and hence the COP are dependent on both the cold side temperature (T_{c}) and the hot side temperature (T_{h}).

For a domestic heat pump, the cold side is the inlet or evaporator side and the hot side the outlet or condenser side. A first-order polynomial expansion of the COP is given by:

$$\mathrm{COP} = c_1 + c_2 T_{\mathrm{in}} + c_3 T_{\mathrm{out}}. \tag{8}$$

A second-order expansion of the COP is more often chosen because it contains an *interaction* term:

$$\mathrm{COP} = \left(c_1 + c_2 T_{\mathrm{in}} + c_3 T_{\mathrm{out}}\right)^2. \tag{9}$$

A model used in [1] describes the heat provided by a heat pump as a second-order expansion of the inlet (T_{in}) and outlet (T_{out}) temperatures:

$$\dot{Q}_{\mathrm{H}} = q_1 + q_2 T_{\mathrm{in}} + q_3 T_{\mathrm{out}} + q_4 T_{\mathrm{in}} T_{\mathrm{out}} + q_5 T_{\mathrm{in}}^2 + q_6 T_{\mathrm{out}}^2. \tag{10}$$

Note that this comes from:

$$\dot{Q}_{\mathrm{H}} = \left(c_1 + c_2 T_{\mathrm{in}} + c_3 T_{\mathrm{out}}\right)^2. \tag{11}$$

Similarly, the electrical power used to drive the heat pump in this model can be expressed as

$$P_{\mathrm{el}} = p_1 + p_2 T_{\mathrm{in}} + p_3 T_{\mathrm{out}} + p_4 T_{\mathrm{in}} T_{\mathrm{out}} + p_5 T_{\mathrm{in}}^2 + p_6 T_{\mathrm{out}}^2. \tag{12}$$

This model, called the YUM model is the basis of the heat-pump model in TRNSYS (Model 401).

2 Fitting heat-pump benchmarks or specs

We developed a Python function to fit the coefficients in Equations 10 for the heating power and 12 for the electrical power to the data from a Hitachi spec sheet (and, while we were at it, we did the same for the cooling power). A selection of fit results are shown in Figures 1 and 2.

Using this code, we obtained analogous results for five more models from the Hitachi Highly series of air–water heat pumps. The typical fit accuracies lie between 0.5 and 1% for P_{el} and P_{heat} (with means of 0.73% and 0.76%, respectively).[1] This results in an accuracy for the resulting COP of $\sim 1.05\%$.

3 Computing the COP and selecting a 'generic' heat pump

We can compute the COP as a function of the two temperatures, by dividing our expressions for P_{heat} and P_{el}. This is shown in Figure 3.

From Figure 3, we have determined the mean COP, where the COP values for each heat-pump type and outlet temperature were averaged over the evaporation temperatures. The result is shown in Fig. 4. We conclude that the mean COPs are fairly constant, and choose the 4 kW model (WHP3970) as a "generic air-water heat pump". We will assume that the power of the generic heat pump can be scaled with the nominal power listed in the specifications for this heat-pump type and use this to compute the heat-pump COP for given temperatures in our numerical model.

[1] And 0.8–1.3% for P_{cool} (mean: 1.07%).

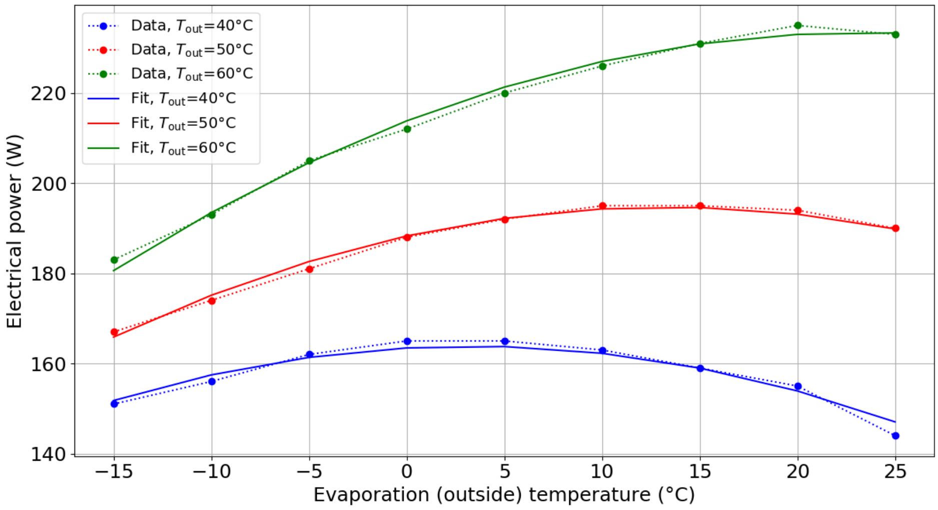

Figure 1: Fitting results for the electrical power needed by the Hitachi Highly WHP00680, as a function of inlet (outside, air) and outlet (inside, water) temperatures. Symbols (with dotted lines) show the data, solid lines the fits.

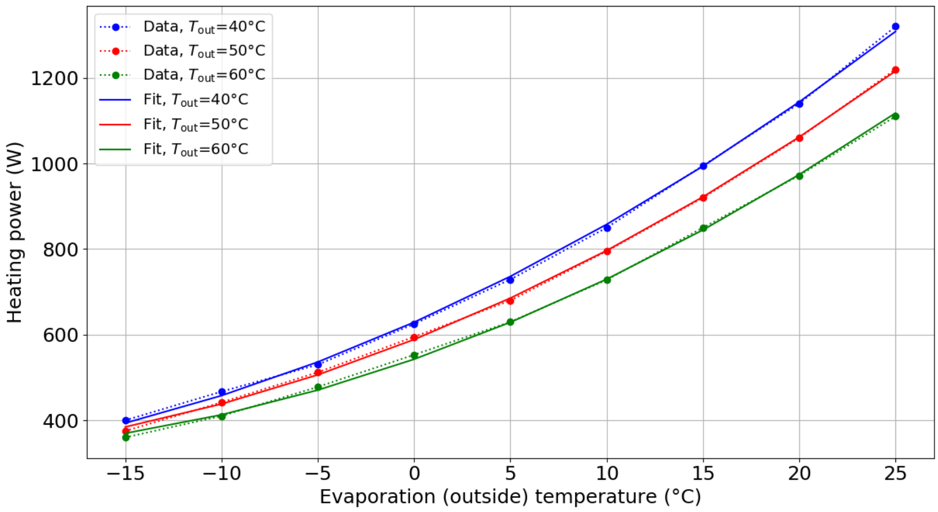

Figure 2: Fitting results for the heating power produced by the Hitachi Highly WHP00680, as a function of inlet (outside, air) and outlet (inside, water) temperatures. Symbols (with dotted lines) show the data, solid lines the fits.

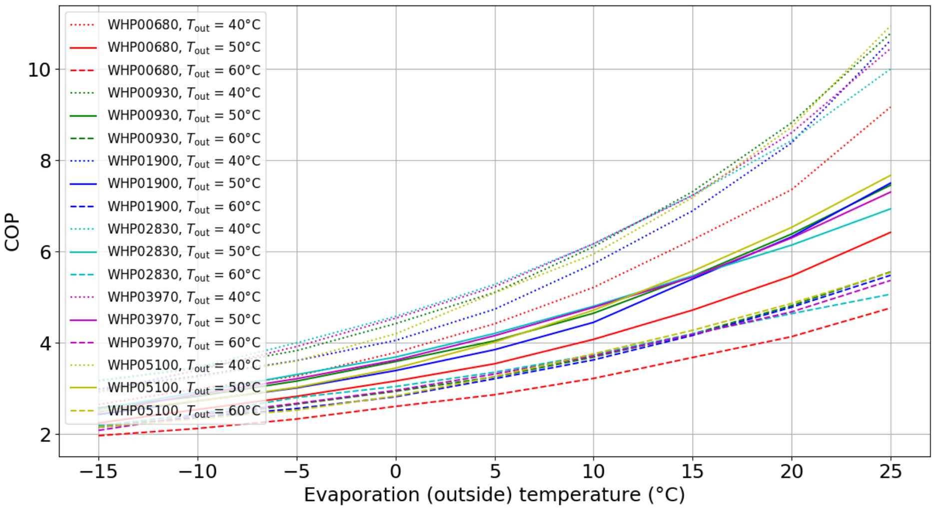

Figure 3: COP of the six Hitachi Highly heat pumps, as a function of inlet and outlet temperatures, as described by our fits. The line styles indicate the outlet (water) temperature, the line colour is different for each heat-pump model. The number in the model name is an indication of the (nominal?) heating power.

4 Defrosting

The model/fit from Sections 2 and 3 is a *static-state* model. Transient effects from switching on or off the heat pump or defrosting are not taken into account. This section describes a simple correction to the COP that takes into account defrosting, following [1].

That paper shows a figure with 11 data points without uncertainty, and a fit using the sum of a Gaussian curve and a straight line, as well as the separate fits for the Gaussian and the line. We have reconstructed their figure in our Fig. 5. The function used is

$$\Delta\text{COP} = a \exp\left(\frac{T-\mu}{\sigma}\right)^2 + bT + c, \tag{13}$$

where T is the outside air temperature in deg C, a, μ and σ are the the amplitude, mean and standard deviation of the Gaussian, respectively, and b and c are the coefficients that describe the line.

We fit both the 11 data points from [1] using a χ^2 test,[2] as well as the Gaussian line and the straight line (assuming that fitting them separately will give a more accurate result than fitting their sum). The resulting fitting coefficients are listed in Table 1. Note that the errors on the coefficients are relatively large. Also note that errors on the fit coefficients for the fits to the lines are meaningless and hence omitted.

The most striking result from the Table is that the two sets of coefficients differ considerably (though due to the large errors not always significantly). This can mean any of three things. Either there are many more original data points than shown, the original data points had varying uncertainties (weights), or something went wrong with the original fit. The results of the two fits are also presented in Fig. 5.

The fit to the data points with assumed uncertainties $\sigma_{\Delta\text{COP}} = 1$ yields a reduced $\chi^2 \approx 3.38$. If the fitting function were a perfect description of the actual behaviour, and the weights were

[2] Since no uncertainties are provided, we assume the same weight for all data points.

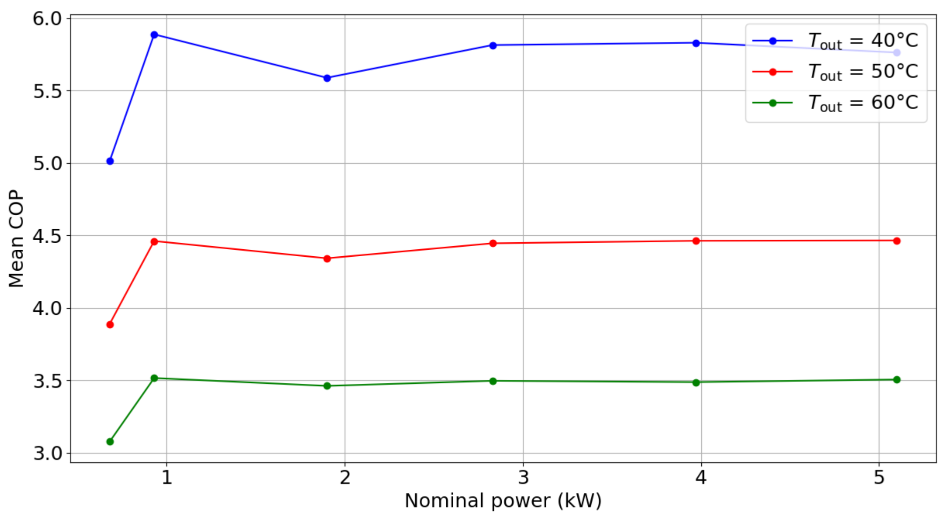

Figure 4: Mean COP as a function of (nominal?) power derived from the heat-pump type name. The COP values from Figure 3 have been averaged over evaporation temperature. Apart from the 680 W model, the mean COPs are remarkably constant.

coef.	**Fit to data points**			**Fits to lines**	**Unit**
	value	**error**	**rel. error**	**value**	
μ	0.948	0.395	41.6%	0.668	°C
σ	3.052	0.557	18.3%	3.951	°C
a	17.197	2.412	14.0%	14.491	%
b	-0.424	0.142	33.5%	-0.410	%/°C
c	3.075	2.090	68.0%	2.785	%

Table 1: Results for the fits to the defrosting data points and lines.

indeed equal, this suggests that $\sigma_{\Delta COP} \approx 1.8\%$. (percentage point, since ΔCOP is expressed in percent).

References

[1] **Afjei, T. & Wetter, M.** *Compressor heat pump including frost and cycle losses*, 1997.

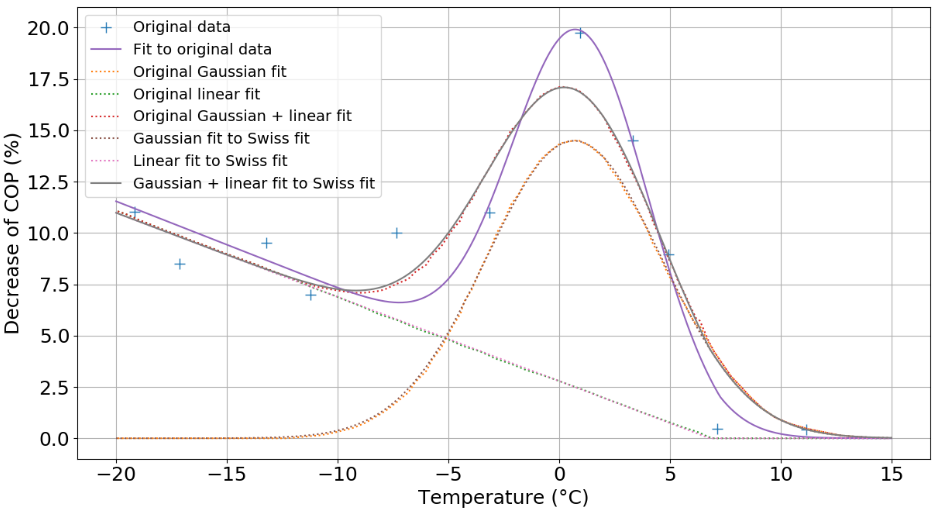

Figure 5: Fits of a Gaussian + linear function to the (original?) data points (solid, purple line fitted to blue crosses) and to the fit line from [1] (grey solid line, fitted to the red dotted line). We find a remarkable difference between the two solid lines.